

# Revision guide

Inferential Statistics · Jérémy Hervelin

**Final exam: 18 December 2026, 13:00–14:00**

Duration: 60 minutes. This guide supports revision; permitted examination aids will be specified separately.

## A revision route

1. **Lectures 1–3:** identify the variable, distribution, event and units.
2. **Lectures 4–5:** explain the sampling design and sampling distribution; distinguish SD and SE.
3. **Lectures 6–9:** estimate a parameter, quantify uncertainty, formulate hypotheses and interpret evidence.
4. **Lectures 10–12:** read R output; distinguish one sample, pairing and independent groups.
5. **Lectures 13–14:** build count tables, check expected counts and select categorical methods.

## Essential formulas and their conditions

| Question                      | Formula                                                                                  | Condition / interpretation                                         |
|-------------------------------|------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| Discrete expectation          | $E(X) = \sum x p(x)$<br>$\text{Var}(X) = E(X^2) - E(X)^2$                                | Sum over support; required moments must exist.                     |
| Binomial count                | $P(X=k) = \text{choose}(n,k) p^k (1-p)^{n-k}$<br>$E(X) = np$ ; $\text{Var}(X) = np(1-p)$ | Fixed $n$ , independent trials, common $p$ .                       |
| Standardise a normal variable | $Z = (X - \mu) / \sigma$                                                                 | $X$ normal; $\sigma$ is the SD, not the variance.                  |
| SE of a mean                  | $\text{SE}(X) = \sigma / \sqrt{n}$ ; estimated $\text{SE} = s / \sqrt{n}$                | Independent identically distributed observations, finite variance. |

| Question                             | Formula                                                                      | Condition / interpretation                                                                                          |
|--------------------------------------|------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| Known-SD mean interval               | $\bar{x} \pm z_{1-\alpha/2} \sigma/\sqrt{n}$                                 | Exact under iid normal sampling and known $\sigma$ ; otherwise possible large-sample approximation.                 |
| Unknown-SD mean interval             | $\bar{x} \pm t_{n-1, 1-\alpha/2} s/\sqrt{n}$                                 | Exact under iid normal sampling. For paired data, use the differences.                                              |
| Approximate proportion interval      | $\hat{p} \pm z_{1-\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$                     | Wald approximation; problematic near boundaries or with small n. Wilson/exact alternatives are discussed.           |
| Mean test                            | $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$                                         | t with n-1 df under the iid normal null.                                                                            |
| Welch comparison                     | $t = [\bar{x}_1 - \bar{x}_2 - \Delta_0] / \sqrt{[s_1^2/n_1 + s_2^2/n_2]}$    | Independent groups; approximate Welch df. Does not require equal variances.                                         |
| Prediction of one future observation | $\bar{x} \pm t_{n-1, 1-\alpha/2} s\sqrt{(1+1/n)}$                            | Independent future draw from the same normal population.                                                            |
| Pearson chi-square                   | $X^2 = \sum (O-E)^2/E$<br>$E_{ij} = \text{row}_i \times \text{column}_j / N$ | Independent count observations and adequate expected counts. Table df = (r-1)(c-1).                                 |
| Goodness of fit                      | $E_j = np_j$ ; df = k-1                                                      | k categories with prespecified probabilities and no estimated parameters. Fitting parameters requires further care. |

## Six representative exam-style questions

Use these as separate practice prompts. They do not specify the final exam's questions, marks or permitted aids.

### ▼ 1 • Model choice (Lectures 1–3)

Four of ten applicant files concern vocational graduates. Three distinct files are selected at random. Find the probability that exactly two are vocational.

**Worked answer:** Hypergeometric sampling without replacement.

$P = \text{choose}(4,2)\text{choose}(6,1)/\text{choose}(10,3) = 36/120 = 0.30$ . A binomial model would ignore the changing success probability.

## ▼ 2 · Sampling uncertainty (Lectures 4–5)

A population has SD 12. Compare the SE of the sample mean for  $n=36$  and  $n=144$  independent observations.

**Worked answer:**  $12/\sqrt{36}=2$  and  $12/\sqrt{144}=1$ . Four times as many observations halves the SE. This does not remove selection bias.

## ▼ 3 · An interval and its interpretation (Lectures 6–7)

Synthetic earnings:  $n=100$ , mean €1800 and known population SD €400. Use  $z=1.96$  for a 95% mean interval.

**Worked answer:**  $SE=40$ ;  $\text{margin}=78.4$ ; interval [€1721.60, €1878.40]. Under the normal independent-sampling model, 95% of repeated intervals cover the fixed mean. It is not an interval containing 95% of individual earnings.

## ▼ 4 · A one-sample decision (Lectures 8–10)

Synthetic search duration:  $n=25$ , mean 5.4 months, SD 2. Test mean 6 against a prespecified lower alternative. The lower 5% t critical value with 24 df is  $-1.711$ .

**Worked answer:**  $t=(5.4-6)/(2/5)=-1.5$ . It is not below  $-1.711$ , so fail to reject at 5%. Do not conclude the mean equals 6; question the normal model for possibly skewed durations.

## ▼ 5 · Design and interpretation (Lectures 11–12)

Scores are observed before and after training for the same 30 workers. A significant positive mean change is found. Which test fits the pairing, and what causal claim is justified?

**Worked answer:** A paired t test is a one-sample t test of after-minus-before differences, exact for independent normal differences across workers. Positive change alone does not establish training impact; time trends or other changes may explain it.

### ▼ 6 · Expected counts (Lectures 13–14)

In a three-by-two table, one row total is 60, one column total is 80 and  $N=200$ . Find that cell's expected count under independence and the table's df.

**Worked answer:**  $E=60 \times 80 / 200 = 24$ ;  $df=(3-1)(2-1)=2$ . Inspect all expected counts and the independence of observations before using the chi-square approximation.

## Common interpretation errors

- A p-value is not the probability that the null is true.
- A confidence level describes a procedure's coverage, not a post-data probability for a fixed parameter.
- Non-significance does not prove equality; significance does not prove practical importance.
- Association, before-after change and random assignment are different bases for interpretation.
- Percentage points, relative percentages, SD and SE are different quantities.
- Choose a one-sided direction before seeing the result.

## Your self-assessment

For each question: can you name the target, justify the model, show the calculation, interpret the result and state a limitation? The website records practice completion when each module reaches at least 4/5. These formative exercises do not contribute to the examination grade.