

Inferential Statistics

Lecture 9: One-sample inference

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Adapted from the teaching materials of Prof. Dr. Thilo Klein.

Our 90-minute route

- First 15 minutes: retrieve prerequisites and introduce the question.
- Next 35 minutes: worked examples, with short checks after each concept.
- Next 25 minutes: paired activity and interpretation.
- Final 15 minutes: synthesis, questions and an applied discussion.

One sample: choose a target and a model

- Test a population mean with a one-sample t test.
- Connect a coefficient t test to simple linear regression.
- Use an exact sign test for a median under a continuous model.
- Explain why tests of a mean and a median answer different questions.

The one-sample t statistic

For $H_0 : \mu = \mu_0$, define

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}.$$

Under iid normal observations, $T \sim t_{n-1}$ under the null. For a two-sided test, reject if $|T| > t_{n-1, 1-\alpha/2}$.

For a one-sided alternative, choose the corresponding single tail before inspecting the observations.

Classroom check – Q1

Exam-style

1–2 minutes

$n=25$, mean=5.4, sample SD=2, null mean=6. Find the one-sample t statistic.

Think individually, then compare your reasoning with a neighbour.

Lecture 9, classroom quiz Q1. Same question number as the printed quiz.

Original jewellery example

The original 20 ratios of length to width have mean 0.6605. Test

$$H_0 : \mu = 0.618, \quad H_1 : \mu \neq 0.618.$$

The sample gives $t = 2.0545$ with 19 degrees of freedom and two-sided $p = 0.05394$.

At 5% we fail to reject. The 95% confidence interval is $[0.6172, 0.7038]$; it narrowly includes 0.618.

Classroom check – Q4

Exam-style

1–2 minutes

Jewellery mean test has $p=0.05394$. At $\alpha=0.05$...

- A. Reject;
- B. Fail to reject; 0.618 remains in the matching 95% CI;
- C. Prove the mean is 0.618

Think individually, then compare your reasoning with a neighbour.

Lecture 9, classroom quiz Q4. Same question number as the printed quiz.

Model checks support interpretation

- Ask how observations were sampled and whether they are independent.
- Plot the distribution and inspect unusual values.
- The exact t reference distribution assumes normal observations.
- Large-sample robustness depends on the population and sampling design; a rule such as n greater than 30 is not a guarantee.

A synthetic job-search example

Suppose $n = 25$ independent normally distributed search-duration observations have $\bar{x} = 5.4$ months and $s = 2$ months.

$$H_0 : \mu = 6, \quad H_1 : \mu < 6, \quad t = \frac{5.4 - 6}{2/\sqrt{25}} = -1.5.$$

Compare with the lower 5% t_{24} critical value, about -1.711 .

We do not reject at 5%. Discuss whether normally distributed durations are plausible and what a skewed distribution would change.

Regression: a conditional mean relationship

Simple linear regression writes

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i.$$

The slope describes the change in the conditional mean of Y for a one-unit change in x under the linear model.

A fitted association between training hours and interview scores is not automatically a causal training effect.

Estimate a regression slope and its uncertainty

With $S_{xx} = \sum_i (x_i - \bar{x})^2$,

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{S_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

Let $s_e^2 = \sum_i (y_i - \hat{y}_i)^2 / (n - 2)$. Then

$$\text{SE}(\hat{\beta}_1) = s_e / \sqrt{S_{xx}}.$$

This standard error assumes independent errors with a common variance and a correctly specified conditional mean.

The coefficient t test

For $H_0 : \beta_1 = b_0$,

$$T = \frac{\hat{\beta}_1 - b_0}{\text{SE}(\hat{\beta}_1)}.$$

With fixed predictors, independent normal errors of mean zero and common variance, this has a t_{n-2} distribution under the null.

The same logic as a mean test applies: estimate minus benchmark, divided by its standard error. Degrees of freedom reflect two estimated regression coefficients.

Classroom check – Q5

Exam-style

1–2 minutes

A simple regression with intercept and slope has $n=20$ observations. What df does the classical slope t test use?

Think individually, then compare your reasoning with a neighbour.

Lecture 9, classroom quiz Q5. Same question number as the printed quiz.

Regression exercise: reproduce before interpreting

Synthetic training hours and interview scores:

$$x = (1, 2, 3, 4, 5, 6), \quad y = (3, 4, 5, 5, 7, 8).$$

In R: `summary(lm(y ~ x))`.

Identify the slope, its standard error, degrees of freedom and p-value. Explain the slope's units. This small artificial dataset illustrates the calculation; it is not empirical evidence about a training programme.

When the median is the question

For a continuous population with median m_0 under the null,

$$P(X > m_0) = P(X < m_0) = 0.5.$$

For n independent observations, the number above m_0 follows $\text{Bin}(n, 0.5)$.

This is the sign test. It does not require symmetry, but it uses signs and discards distances. It targets a median rather than a mean.

Classroom check – Q2

Concept check

1–2 minutes

A sign test against a continuous-population median uses...

- A. Distances from the median;
- B. Signs relative to the benchmark;
- C. Squared deviations

Think individually, then compare your reasoning with a neighbour.

Lecture 9, classroom quiz Q2. Same question number as the printed quiz.

Original online-auction prices

Nine prices: 51, 56, 57, 48, 45, 61, 46, 53 and 59.

Against median 50, six are above and three below. For the prespecified alternative median greater than 50,

$$p = P\{\text{Bin}(9, 0.5) \geq 6\} = 0.25390625.$$

Fail to reject at 5%. This does not establish that the expected price equals 50.

Classroom check – Q3

Exam-style

1–2 minutes

6 of 9 auction prices exceed 50. The prespecified alternative is median >50 . Find $P(\text{Bin}(9,0.5) \geq 6)$.

Think individually, then compare your reasoning with a neighbour.

Lecture 9, classroom quiz Q3. Same question number as the printed quiz.

Ties and the sign test

The usual continuous-model calculation assumes no observations equal the hypothesised median.

A common convention removes ties and tests whether the remaining signs are equally likely. With genuine point mass at the benchmark, that conditional sign question need not be equivalent to every possible definition of a median null.

Report ties, the effective sample size, and the tested sign probability. Do not hide ties or silently count them as positive.

The jewellery mean and median tests differ

Against 0.618, 11 of the 20 jewellery observations are above the benchmark. The two-sided exact sign-test p-value is approximately 0.8238.

The mean-test p-value was 0.05394. These tests use different information and target different parameters; neither result is a contradiction of the other.

Activity: explain how an unusually large ratio can affect a mean test much more than a sign test.

Preparation for the R practical

- Write hypotheses before running a command.
- Identify the estimate, confidence interval, statistic, degrees of freedom and p-value.
- Inspect the data and explain the assumptions.
- Write a three-sentence interpretation including magnitude, uncertainty and limitations.

Independent practice

- Practise the five online questions for this lecture; 80% provides access to the next practice module.
- Course materials remain accessible regardless of your practice progress.
- Explain your reasoning before comparing answers.
- Review the method, assumptions and interpretation as well as the calculation.

Sources and teaching notes

Adapted from Prof. Dr. Thilo Klein's supplied Inferential Statistics materials.

Revised teaching edition: Jérémy Hervein, Pforzheim University, winter semester 2026/27.

New youth-employment, programme and interview-score examples are synthetic illustrations.